BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

PRE BOARD-2, (2024-25)

MATHEMATICS (041)

Class: XII Science — Duration: 3 Hour
Date: 11/12/24 — Max. Marks: 80
Admission Number: ___________ — Roll number: _____

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion - Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) - type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA) - type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA) - type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study - based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1 Total number of possible matrices of order 3×3 with each entry 2 or 0 is **[1]**

a) 27

b) 81

c) 9

d) 512

2 If A is a singular matrix, then A(adj A) is **[1]**

a) Scalar matrix

b) Inverse matrix

c) Identity matrix

d) Null matrix

3 Let $\begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix}$ = px ^{7} + qx ^{6} + rx ^{5} + sx ^{4} + tx ^{3} + ux ^{2} + vx + w then which of the following is not true? **[1]**

a) w = 21, v = 75

b) p = - 1, t = 8

c) p = q = - 1

d) q = 0, s = - 4

4 If y = f$\left(\frac{3x+4}{5x+6}\right)$ and f'(x) = tan x ^{2} then $\frac{dy}{dx}$ is equal to **[1]**

a) - 2 tan$\left(\frac{3x+4}{5x+3}\right) \times \frac{1}{(5x+3)^2}$

b) tan x^2

c) $f\left(\frac{3\tan x^2+4}{5\tan x^2+6}\right)$

d) - 2 tan$\left(\frac{3x+4}{5x+6}\right) \times \frac{1}{(5x+6)^2}$

5 If a line makes an angle of 30^o with the positive direction of x - axis, 120 ^{o} with the positive direction of y - axis, then the angle which it makes with the positive direction of z - axis is: **[1]**

a) 0^o

b) 60^o

c) 90^o

d) 120^o

6 What is the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{3/2} = \left(\frac{d^2y}{dx^2}\right)^2$? **[1]**

a) 3

b) 2

c) 1

d) 4

7 A linear programming problem deals with the optimization of a/an: **[1]**

a) logarithmic function

b) exponential function

c) linear function

d) quadratic function

8 If $\vec{a} = \left(2\hat{\imath} + 4\hat{\jmath} - \hat{k}\right)$ and $\vec{b} = \left(3\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}\right)$ be such that $\vec{a} \perp \vec{b}$ *then* $\lambda =$? **[1]**

a) 3

b) - 2

c) 2

d) - 3

9 The primitive of $\frac{2}{1+\cos 2x}$ is **[1]**

a) 2 $\sec^2$ x tan x

b) $\sec^2$ x

c) - cot x

d) tan x

10 A matrix $A = [a_{ij}]_{3\times 3}$ is defined by $a_{ij} = \begin{cases} 2i + 3j & , \quad i < j \\ 5 & , \quad i = j \\ 3i - 2j & , \quad i > j \end{cases}$ **[1]**

The number of elements in A which are more than 5, is 4:

a) 5

b) 6

c) 4

d) 3

11 Which of the following statements is correct? **[1]**

1. Every LPP admits an optimal selection.
2. A LPP admits unique optimal solution.
3. If a LPP admits two optimal solutions it has an infinite solution.
4. The set of all feasible solutions of a LPP is not a convex set.

a) Option (d)

b) Option (a)

c) Option (b)

d) Option (c)

12 If $|\vec{a}|$=10, $|\vec{b}|$= 2 and $\vec{a}.\vec{b}$ =12, the the value of $|\vec{a}\ x\ \vec{b}|$ is **[1]**

a) 5

b) 10

c) 14

d) 16

13 If P is a 3× 3 matrix such that P' = 2P + I, where P' is the transpose of P and I is 3 × 3 identity matrix, then there exists a column matrix X = $\begin{vmatrix} x \\ y \\ z \end{vmatrix} \neq \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$ such that **[1]**

a) PX = 2X

b) PX = $\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

c) PX = - X

d) PX = X

14 A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is **[1]**

a) $\frac{3}{28}$

b) $\frac{1}{14}$

c) $\frac{33}{56}$

d) $\frac{9}{64}$

15 The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is **[1]**

a) $e^x + e^y = \frac{x^3}{3} + c$

b) $e^x - e^y = \frac{x^3}{3} + c$

c) $y = e^{x-y} - x^2 e^{-y} + c$

d) $e^y - e^x = \frac{x^3}{3} + C$

16 The projection of vector $\hat{\imath}$ on the vector $\hat{\imath} + \hat{\jmath} + 2\hat{k}$ is: **[1]**

a) $\sqrt{6}$

b) $\frac{2}{\sqrt{6}}$

c) $\frac{3}{\sqrt{6}}$

d) $\frac{1}{\sqrt{6}}$

17 The function f(x) = [x] , where [x] denotes the greatest integer function, is continuous at **[1]**

a) - 2

b) 1.5

c) 1

d) 4

18 If the points A(- 1, 3, 2), B(- 4, 2, - 2) and C(5, 5, λ) are collinear then the value of λ is **[1]**

a) 5

b) 10

c) 8

d) 7

19 **Assertion (A):** The absolute maximum value of the function $2x^3 - 24x$ in the interval [1, 3] is 89. **[1]**

Reason (R): The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20 **Assertion (A):** $f(x) = 1 + x^2$ is a one to one function from $R^+ \to R$. **[1]**

Reason (R): Every strictly monotonic function is a one to one function.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21 Find the principal value of $\operatorname{cosec}^{-1}$ (- 1). **[2]**

OR

Write the interval for the principal value of function and draw its graph: $\sec^{-1}$ x.

22 Find the maximum and minimum value, f(x) = |x + 2| - 1 **[2]**

23 Find the intervals in which f(x) is increasing or decreasing:f(x) = sin x(1 + cos x), 0 < x <$\frac{\pi}{2}$ **[2]**

OR

Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

24 Evaluate:$\int_0^{\pi} \frac{x}{(a^2\cos^2 x+b^2\sin^2 x)} dx$. **[2]**

25 Prove that the function f(x) = $\log_a$ x is strictly increasing on when a > 1 and strictly decreasing on when 0 < a < 1. **[2]**

Section C

26 Integrate the rational function $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ **[3]**

27 The bag A contains 8 white and 7 black balls while the bag B contains 5 white and 4 black balls. One ball is randomly picked up from the bag A and mixed up with the balls in bag B. Then a ball is randomly drawn out from it. Find the probability that ball drawn is white. **[3]**

28 Evaluate the integral:$\int_0^{\pi/2} |\sin x - \cos x| \, dx$ **[3]**

OR

Evaluate:$\int$ e ^{sinx} sin 2x dx

29 Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{y(2y-x)}{x(2y+x)}$ given that y = 1 when x = 1. **[3]**

OR

Solve the initial value problem:$y - x\frac{dy}{dx} = 2\left(1 + x^2\frac{dy}{dx}\right)$, y(1) = 1

30 Solve the following LPP graphically: **[3]**

Maximize and Minimize Z = 3x + 5y

Subject to 3x - 4y + 12 $\geq$ 0

2x - y + 2 $\geq$ 0

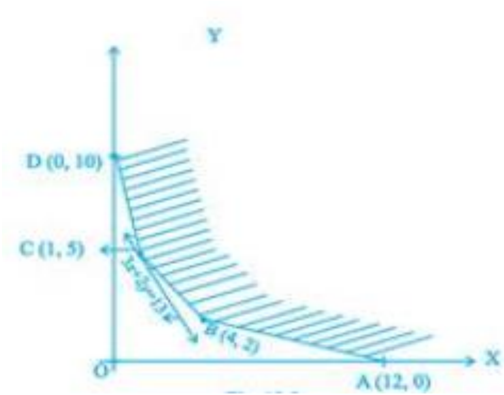
2x + 3y - 12 $\geq$ 0

$0 \leq x \leq 4$

$y \geq 2$

OR

Determine the minimum value of Z = 3x + 2y (if any), if the feasible region for an LPP is shown in Fig.



31 Find all points of discontinuity of f where f is defined as follows, $f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$ **[3]**

Section D

32 Find Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the lines x + y = 2. **[5]**

33 Show that the function f : R - {3}→ R - {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection. **[5]**

OR

Show that the function f: $R_0 \to R_0$, defined as f (x) = $\frac{1}{x}$, is one - one onto, where R_0 is theset non - zero real numbers. Is the result true, if the domain R_0 is replaced by N with co - domain being same as R_0 ?

34 If A = $\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations: **[5]**

3x-4y+2z =-1, 2x+3y+5z =7 and x+z=2.

35 If $l_1, m_1, n_1 ; l_2, m_2, n_2 ; l_3, m_3, n_3$ are the direction cosines of three mutually perpendicular lines, prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ makes equal angles with them. **[5]**

OR

By computing the shortest distance determine whether the pairs of lines intersect or not:

$\vec{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(2\hat{\imath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$

Section E

36 **Read the following text carefully and answer the questions that follow:** **[4]**

In pre - board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

1. Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics? (1)
2. Findthe probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics? (1)
3. Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics? (2)

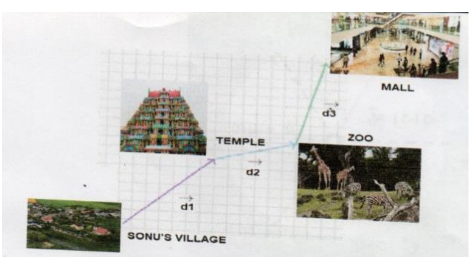
   OR

   Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics? (2)

37 **Read the following text carefully and answer the questions that follow:** **[4]**

Sonu left from his village on weekend. First, he travelled d_1 displacement up to a temple. After this, he left for the zoo and travelled d_2 displacement. After this he left for shopping in a mall - Total driving time of Deepal from village to Mall was 1.5 hr.

If $d_1 = (6, 8)$ $d_2 = (3, 4)$ and $d_3 = (7, 12)$ km



1. What is the total displacement from village to Mall? (1)
2. What is the speed of Sonu from Village to Mall? (1)
3. What is the Displacement from Village to Zoo? (2)

   OR

   What is the displacement from temple to Mall? (2)

38 **Read the following text carefully and answer the questions that follow:** **[4]**

The temperature of a person during an intestinal illness is given by $f(x) = - 0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^{o} F at x days.

1. Is the function differentiable in the interval (0, 12)? Justify your answer. (1)
2. If 6 is the critical point of the function, then find the value of the constantm. (1)
3. Find the intervals in which the function is strictly increasing/strictly decreasing. (2)

   OR

   Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local minimum and the absolute maximum/absolute minimum values of the function. (2)